A Small test – Data based and Knowledge based systems (10.4.2012)

1) Fuzzy set and classical set. Describe differencies !
2) Let us consider universum U = $\{a,b,c,d\}$, on which is defined fuzzy relation equivalence "≈" by the following way:

   $x,y \in U$, $\mu_{\approx}(x,y) = \{0.1/(a \approx b), 0.5/(a \approx d), 0.4/(b \approx c), 0.9/(c \approx d)\}$.

   (Relation equivalence has the property « transitivity » as relation ordering and it is possible to use expressions (12.4), (13.4) and method for computation of relation of order for pairs (a,c), (b,d). Equivalence is symmetric and instead of (15.4) is necessary to use

   $$x,y \in U,\ \mu_{\approx}(x, y) = \mu_{\approx}(y, x) \ .$$

3) Which operation on classical sets corresponds to fuzzy composition rule ?
4) Compute fuzzy inference according to Mandani for (x, 0.55), (y, 0.35).

3. Rule based systems

In artificial intelligence has the term „rule“ a special meaning and significance. The term „rule“ is associated with the structure:

- ⟨IF (sentence, statement, fuzzy relation,…)⟩ ⇒ ⟨THEN (sentence, statement, fuzzy relation,…)⟩,
- ⟨IF (Conditions)⟩ ⇒ ⟨THEN (Consequences)⟩,
- … .

Note 3.1:

- Knowledge as a *property* of someone.
- Knowledge as a symbolical form associated with a *semantic content* („Warning! T= 1000 K“) or with an *action* („Stop the procedure !!“).
- The symbolical form appropriate for a communication (e.g., with the computer) is called *representation of knowledge*.

Syntactic forms of the rule:

- <L, R>. ⟨IF (L is verified as true (consistent, correct)) ⟩ ⇒ ⟨THEN (Is applied R)⟩,
- More detailed structure of the rule :

$$< f(C_1, ..., C_n), (w_1, ..., w_n) > \; ---> \; < D, g(w_1, ..., w_n) >, \qquad (29.4)$$

where $C_1, ..., C_n$ are conditions of the rule, $w_1, ... , w_n$ are weights assigned to conditions, D is a consequence of action of left side, f is an ***interaction function*** (f determines types of interactions of conditions) and $g(w_1,...,w_n)$ is a numerical function for computation of weight of the consequence D.

Example 7.4.: Let us consider a technological systems that works as a heat-exchanger in an environment with disturbances. These disturbances is hard to describe by means of some analytical models. Nevertheless – is possible to collect empirical knowledge of experts and human operators and formalize the behavior of the exchanger by rules:

- 〈IF (The temperature of input heat-exchanging liquid is higher than 429 [°K] (C_1)) AND 〈(The volume of input liquid Q_1 decreases under the value 0.02 [m^3/sec] (C_2)) OR (The volume of output matter Q_2 decreases under the value 0.1 [m^3/sec] (C_3))〉〉 ⇒ 〈THEN (The internal temperature of the system increases above the value 500 [°K] during 9 [min] (D))〉,

The structure of this rule with interaction functions (AND, OR) and with weights of conditions (w_1,w_2,w_3) is in Fig. 9.4. ■

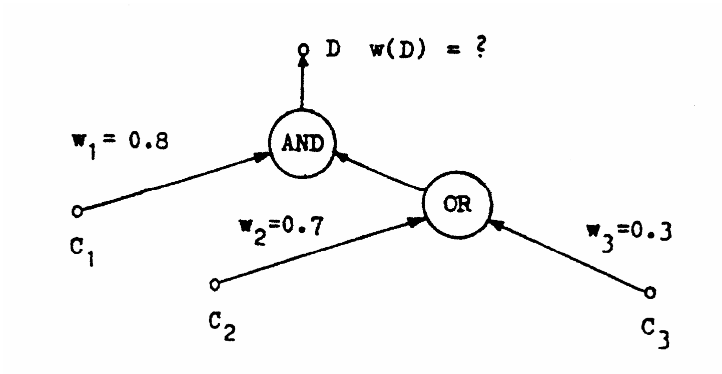

Fig. 9.4

Variables w_1, w_2, w_3 represent internal dependencies in the rule and describe some typical features of the system, e.g., consequences of physical laws. They have similar significance as quotients of importance (usually defined on the interval 〈0,1〉 and the larger value, the more importance). These variables are determined by an expert that knows completely behavior of the system. W_1, w_2, w_3, however do not describe actual situations in the modeled system. They are usually called „aposteriori weights“. Actual situations in the system express as a fulfillment of C_1, C_2, C_3 weights μ_{Ci}(…), e.g., μ_{C1}(T_1=311[°K]) = 0.71. The conditions together with their weights are fuzzy sets defined on the proposed universes.

Computations of outputs of interaction functions use so called „actualized weights“ that are usually expressed as product

$$w_{ia} = \mu_{Ci} * w_i \text{ , for } i = 1, \ldots, n \qquad (30.4)$$

The values of actualized weights input into interaction functions of the rule. Outputs of the interaction functions are computed according to fixed forms, e.g.,:

- f = AND, $w(D) = g(w_{1a},...,w_{na}) = \min w_{1a},...,w_{na}\}$, (31.4)
- f = OR, $w(D) = g(w_{1a},...,w_{na}) = \max \{w_{1a},...,w_{na}\}$, (32.4)
- f = NOT, $w(D) = g(w_{ia}) = 1 - w_{ia}$, (33.4)
- f = H_M , $w(D) = g_M(w_{1a}, g_M(w_{2a},...,w_{na})) = g_M(w_{1a}, g_M(w_{2a}, g_M(w_{3a},...,w_{na}))) =$
  $= g_M(w_{1a}, g_M(w_{2a}, g_M(w_{3a}, ..., g_M(w_{n-2,a}, (w_{n-1,a}+w_{na} - w_{n-1,a} * w_{na}))))...)$. (34.4)
- f = H_P , $w(D) = g_P(w_{1a}, g_P(w_{2a},...,w_{na})) = g_P(w_{1a}, g_P(w_{2a}, g_P(w_{3a},...,w_{na}))) =$
  $= g_P(w_{1a},g_P(w_{2a}, g_P(w_{3a}, ..., g_P(w_{n-2,a}, (w_{n-1,a}+w_{na}/(1+ w_{n-1,a}* w_{na})))))...)$. (35.4)

Note 3.2: Computations for special functions H_M a H_P are derived from „fathers“ of expert systems – PROSPECTOR (geological research) and MYCIN (diagnostics of blood).

Note 3.3: In case when output of some interaction function $w({}^{j-1}D)$ input as a condition in the following interaction function it is usually considered

$$w({}^{j}C_i) = w({}^{j-1}D).$$

Example 8.4: Compute the weight of the consequence w(D) for the rule introduced in Fig.9.4, with values $\mu_{C1} = 0.7$, $\mu_{C2} = 0.9$ a $\mu_{C3} = 0.8$.

At first we compute the weight of the consequence D_1 of the rule OR then the weight of consequence D :

$$w(D_1) = \max \{w_{2a}, w_{3a} \} = \max \{0.63, 0.24 \} = 0.63 .$$

$$w(D) = \min \{ w_{1a}, w_a(D_1) \} = \min \{0.56, 0.63 \} = 0.56 .$$

The weight w(D) is interpreted as a fact that in the actual state of the system the certainty of condition „The internal temperature of the system increases above the value 500 [°K] during 9 [min] (D))〉" is 56 %. ■

Recommended and used literature:

[1] Kruse, R., Gebhardt, J. and Klawonn, F.: Foundation of Fuzzy Systems. John Wiley & Sons Ltd., Chichester, England, 1994.

[2] M. Delgado, N. Marín, D. Sánchez, María-Amparo Vila: Fuzzy Association Rules: General Model and Applications. *IEEE Trans. On Fuzzy Systems* 2/11, 2003, pp. 214-226.

[3] Bíla, J., Šmíd, J., Král, F. a Hlaváč, V.: Informační technologie. Databázové a znalostní systémy. ČVUT v Praze, 2009.